Foundations 20 Final Exam Review - Answer Key

1. True or False.

a) F a conjecture is a testable expression that is NOT proven to be true.
b) T
c) F a counter example INVALIDATES a conjecture.
d) T
e) T
f) T
g) F The more support you have for a conjecture, the stronger it is.
h) F You must prove your conjecture true for all cases
i) F
j) T
k) T
l) F Circular reasoning is NOT a reliable way to prove your conjecture

2. odd + even = sum

collect evidence
- 3 + 4 = 7
- 5 + 6 = 11
- 21 + 22 = 43
- -3 + 6 = 3

make a conjecture:
The sum of one odd integer and one even integer will result in an odd integer

test your conjecture.
- 15 + 18 = 33
- 101 + 100 = 201

3. $(odd)^2$ = odd
- $(3)^2 = 9$
- $(-5)^2 = 25$
- $(11)^2 = 121$
- $(1)^2 = 1$

Yes, Paula's conjecture is reasonable.

4.

Choose #	6	10
× 2	12	20
+ 6	18	26
× 2	36	52
- 4	32	48
÷ 4	8	12
- 2	6	10

conjecture:
The # you end up with was the # you started with.

5.

n	
2n	
2n + 6	
4n + 12	
4n + 8	
n + 2	
n	

6. (odd integer)^2 - (the odd integer) = even.

inductive examples:

a) $(3)^2 - (3) =$
$9 - 3 = 6$

b) $(5)^2 - 5$
$25 - 5 = 20$

deductive PROOF: Let $2x+1$ = any odd integ

$(2x+1)^2 - (2x+1) =$ even
$4x^2+4x+1 - (2x+1) =$ divisible by two
$4x^2+4x+1-2x-1 =$
$4x^2 + 2x =$
$2(2x^2+x) =$ ✓ divisible by two means its even.

7. Khaki

8.
2	7	6
9	5	1
4	3	8

2	9	4
7	5	3
6	1	8

9. Andy Bonnie Candice Darlene

Scenarios: A C B } based on clue #1
B C A }

based on clue #3 → * D A C B } based on clue #2
B C A D }

Darlene, Andy, Candice, Bonnie.

10. a) 180°
b) equal
c) transversal
d) Interior
e) outside
f) corresponding
g) alternate interior, =
h) Supplementry

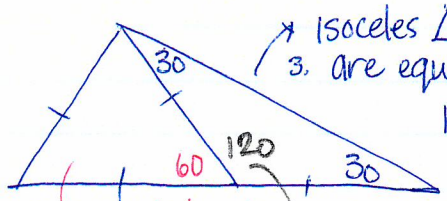
11. In order to prove that two lines are //
1) Corresponding angles must be =.
2) alternate interior angles must be =.
3) alternate exterior angles must be =.
4) Same side interior angles must be supplementry

12. Vertically opposite angles being = DOES NOT prove that two lines are //.

13. a) Corresponding angles are =, so lines are //
b) Same side interior angles are NOT supplementry, so line are NOT //.
c) Alternate exterior angles are =, so lines are //.
d) same as c)

14. a) $\angle a = 75°$ b/c corresponding $\angle$
$\angle c = 105°$ b/c supplementry to 75°/definition of a line.
$\angle d = 105°$ b/c vertically opposite to c + supplementry to 75°
$\angle b = 105°$ b/c corresponding to c + alternate exterior to d

15.



3. Isoceles $\triangle$ so base angles are equal.

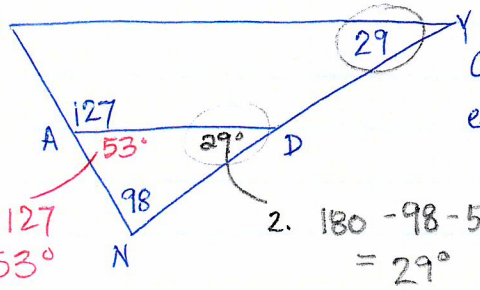
$180 - 120 = 60$ $\quad \frac{60}{2} = 30$

$\angle A = 30°$

1. equilateral so all angles = 60°

2. $180 - 60 = 120$ def'n of a line

16.



corresponding $\angle$'s are equal so $SY \parallel AD$.

1. $180 - 127 = 53°$ def'n of a line

2. $180 - 98 - 53 = 29°$

18. $\angle y = 180 - 40 - 45$ } interior angles of a $\triangle$
$y = 95°$

$\angle x = 40°$ } line $\parallel$ so alt int $\angle$'s are =
$\angle z = 45°$

17. lines are $\parallel$ so,

a) $\angle c = 76°$ (vertically opposite)
$\angle b = \angle c$ = alternate exterior
$\angle a = 180 - 76 = 104°$

b) $2a + 3a = 180$ (same side int)
$\frac{5a}{5} = \frac{180}{5}$
$a = 36°$
$b = 3a$ $\quad c = b$ (alt exterior)
$b = 3(36)$ $\quad c = 108°$
$b = 108°$

19. $180(n-2) =$
$180(20-2) =$
$180(18) = 3240°$

20. $\frac{180(n-2)}{180} = \frac{3060°}{180}$
$n - 2 = 17$
$+2 \quad +2$
$n = 19$ sides

21.a) $\frac{180(n-2)}{n} = 140°$

$180(n-2) = 140n$

$180n - 360 = 140n$

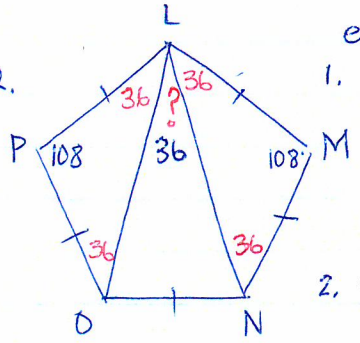
$\frac{-360}{-40} = \frac{-40n}{-40}$

$9 = n$

b) if the interior angle is 140°, the one exterior angle is 40°

$9 \times 40° = 360°$

22.



each interior ∠.

1. $\frac{180(5-2)}{5} = 108°$

2. $180 - 108 = 72$

$\frac{72}{2} = 36°$

3. $108 - 36 - 36 =$

$= 36°$

23. exterior ∠'s of a convex polygon have a sum of 360°

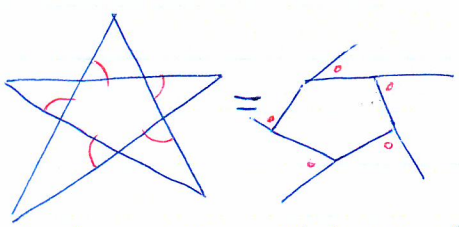
$70 + 60 + 90 + x + x = 360$

$2x = 140$

$x = 70°$

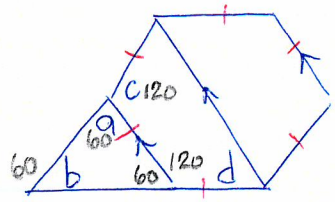
The interior angles would then be 110, 110, 90°, 110°, 110°

24.



exterior angles add to 360°

25. a)



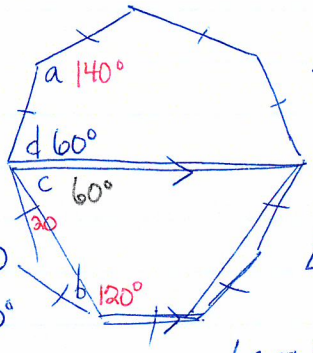
$\angle c = \frac{180(6-2)}{6} = 120°$

$\angle a = 180 - 120° = 60°$

$\angle b = 60°$

$\angle d = 60$ corresponding to 60°

b)



$\frac{180(9-2)}{9}$

$140°$

$\angle a = 140°$

$180 - 140 = 40$

$\frac{40}{2} = 20°$

$\angle b = 20°$

$\angle c = 180 - 120$

$\angle c = 60°$

$\angle d = 140 - 20 - 60$

$\angle d = 60°$

26.a) $2x + 50 + x + 35 + 2x = 180$

$5x + 85 = 180$

$\frac{5x}{5} = \frac{95}{5}$ $x = 19$

b) $130 = 3x + 2x$

$130 = 5x$ exterior angle is

$26 = x$ equal to the sum of the two interior non-adjacent angles

27. $\sin 50 (8.0) = \sin 60 (w)$

a) $\frac{6.1.28}{.8660} = \frac{.8660(w)}{.8660}$

$7.1 = w$

b) $\sin 72 (6) = \sin M (10)$

$\frac{5.7}{10} = \frac{\sin M (10)}{10}$

$.5706 = \sin M$

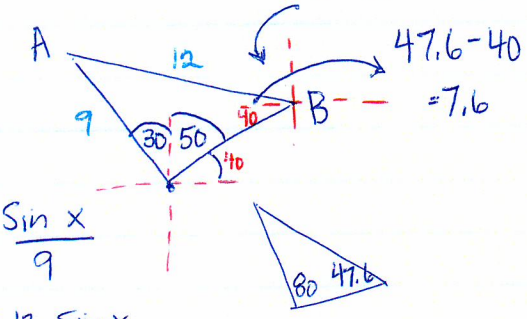
$34.8° = M$

28. $\frac{\sin 80°}{12} = \frac{\sin 55°}{x}$

$\sin 55 (12) = \sin 80 (x)$

$10 = x$

29.



$\frac{\sin 80}{12} = \frac{\sin x}{9}$

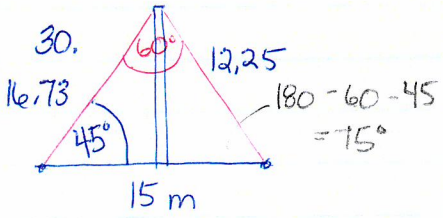
$8.863 = 12 \sin x$

$.7386 = \sin x$

$47.6° = x$

The captain should head N82° W from light house B

30.

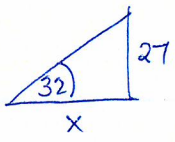


$\frac{\sin 60}{15} = \frac{\sin 45}{x}$ $\frac{\sin 60}{15} = \frac{\sin 75}{x}$

$x = 12.25$ $x = 16.73$

height = $\sin 45 = \frac{h}{16.73} = 11.83$

31. a)



$\tan 32 = \frac{27}{x}$

$.6249 = \frac{27}{x}$

$\frac{27}{.6249} = x$

$43.2 = x$

b) $\tan 43 = \frac{h}{43.2}$

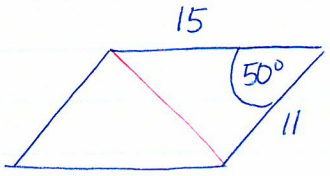
$.9325 (43.2) = h$

$40.3 = h$

height of crane = $40.3 - 27$

$= 13.3$ m

32.



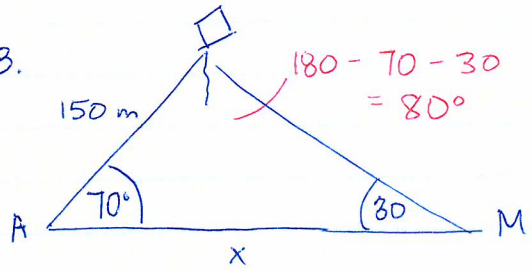
$a^2 = 15^2 + 11^2 - 2(15)(11)(\cos 50)$

$a^2 = 225 + 121 - 212.$

$a^2 = 134$

$a = 11.6$

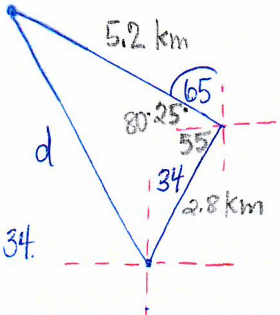
33.



$\frac{\sin 30}{150} = \frac{\sin 80}{x}$

$\frac{147.7}{.5} = \frac{.5x}{.5}$

$295.4 = x$



$d^2 = (5.2)^2 + (2.8)^2 - 2(5.2)(2.8)\cos 80$

$d^2 = 29.8$

$d = 5.5$ km = distance

$$\frac{\sin 80}{5.5} = \frac{\sin x}{5.2}$$

$$\frac{5.121}{5.5} = \frac{5.5(\sin x)}{5.5}$$

$.9311 = \sin x$

$68.6^\circ = x$

$68.6 - 34 = 34.6^\circ$

N 35 W = direction

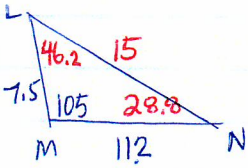
35. a) .64 ⇝ 40° and 140°

b) $\frac{1}{3}$ ⇝ 19.5° and 160.5°

c) .95 ⇝ 71.8° and 108.2°

d) $\frac{7}{23}$ ⇝ 17.7° and 162.3°

36. a)



$$\frac{\sin 105}{15} = \frac{\sin L}{11.2}$$

$\angle L = 46.2^\circ$

$x^2 = 7.5^2 + 11.2^2 - 2(7.5)(11.2)(\cos 105)$

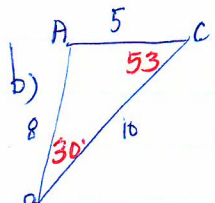
$x^2 = 225.2$

$x = 15$

$$\frac{\sin 105}{15} = \frac{\sin N}{7.5}$$

$28.8 = \angle N$

b)



$$\cos B = \frac{5^2 - 10^2 - 8^2}{-2(10)(8)}$$

$\cos B = .86875 = 30^\circ$

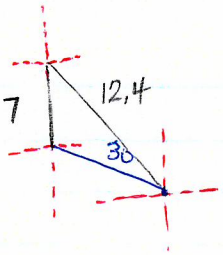
$$\frac{\sin 30}{5} = \frac{\sin C}{8}$$

$53^\circ = C$

$\angle A = 180 - 30 - 53$

$= 97^\circ$

37.



$$\frac{\sin 30}{7} = \frac{\sin L}{12.4}$$

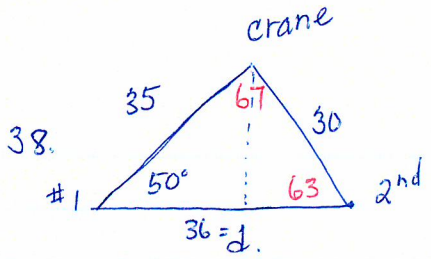
$.8857 = L$

$62.4 = L$ or $180 - 62.4 = 117.6^\circ$

$180 - 118 = 62^\circ$

Compass direction S 62°E will get them back to the parking lot.

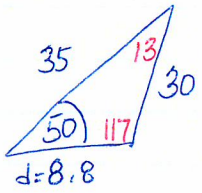
38.



SSA situation!

$$\sin 50 = \frac{\text{height}}{35}$$

$26.8 = h$



2 possible triangles

$$\frac{\sin 50^\circ}{30} = \frac{\sin x}{35}$$

$.8937 = \sin x$

$63^\circ = x$ OR 117° ($180 - 63$)

$$\frac{\sin 50^\circ}{30} = \frac{\sin 67}{x}$$

$36 = x$

$$\frac{\sin 50}{30} = \frac{\sin 13}{x}$$

$8.8 = x$

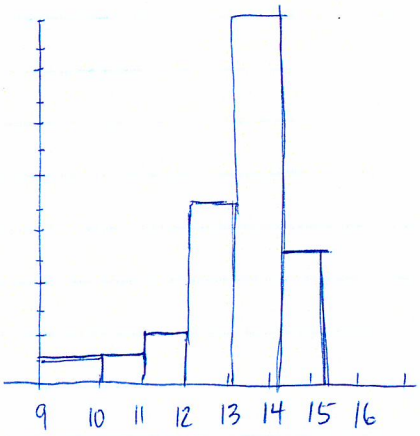
The two workers are either 36 m or 8.8 m apart.

9.6, 10.4, 11.5, 11.8, 12.8, 12.8, 12.8, 12.9, 12.9, 12.9, 13, 13.1, 13.1, 13.2, 13.2, 13.3, 13.3, 13.4, 13.5, 13.5, 13.6, 13.6, 13.6, 13.8, 13.9, 14.1, 14.4, 14.5, 14.6, 14.8

39. mean = 13.13
median = 13.25
mode = 12.8, 12.9 + 13.6

range = 14.8 − 9.6 = 5.2
σ = 1.1007

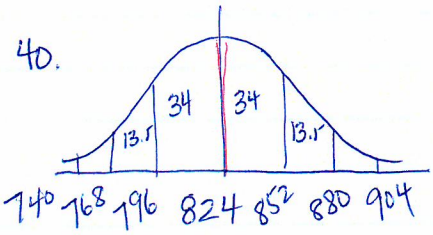
interval	tally	freq
9.0 - 10.0	I	1
10 - 11.0	I	1
11.0 - 12.0	II	2
12 - 13.0	IIII II	7
13 - 14.0	IIII IIII IIII	14
14 - 15.0	IIII	5



d) The range & σ are low which tells us that the data is not spread out too far and is relatively consistent.

c) the data is somewhat normally distributed.

40.

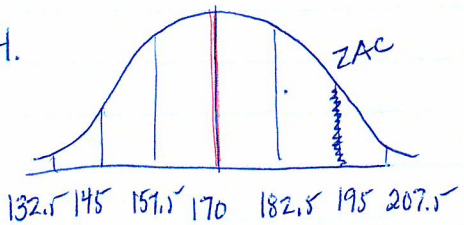


b) 34 + 34 = 68%
796 824 852

c) 2.35 + 13.5 + 34 = 15.85%
740 768 796 824

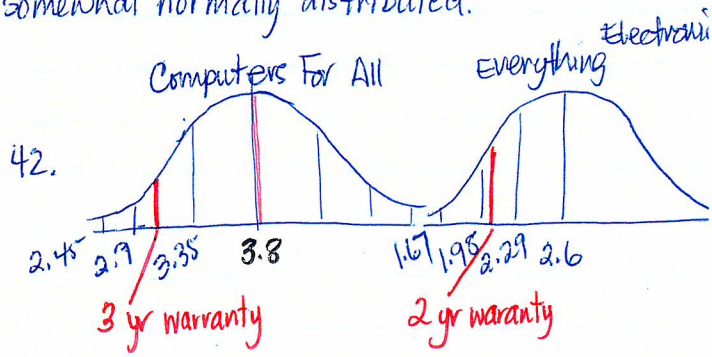
d) 95% = between 768 km & 880 km

41.



a) 50 + 34 + 13.5 = 97.5% shorter
b) 100 − 97.5 = 2.5% taller.

42.



$Z = \frac{3 - 3.8}{.45}$ $\quad Z = \frac{2 - 2.6}{.31}$

$Z = -1.78$ $\quad Z = -1.94$

= use your table =

.0375 → 3.75% $\quad$ .0262 → 2.62%

Computers For All is more likely to fail before its warranty is over because there is more area to the left.

43. Female: $Z = \frac{277 - 247}{33} = .9091$

Male: $Z = \frac{499 - 461}{51} = .7451$

The female had the greatest mass when compared to the other bears b/c its weight is further from the average.

44. a) margin of error = ± 3.1%

b) confidence level = $\frac{19}{20}$ or 95%

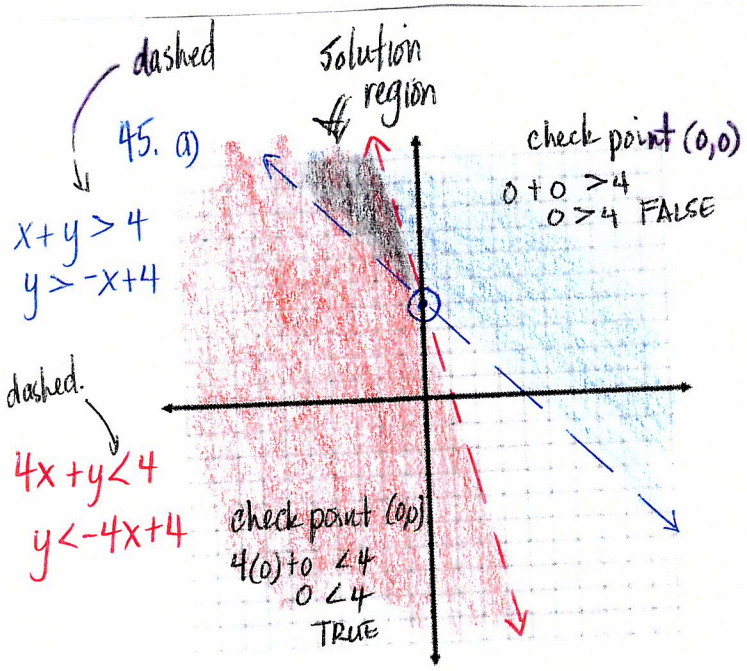
c) i) health care = 23.1 ± 3.1%

20% – 26.2%

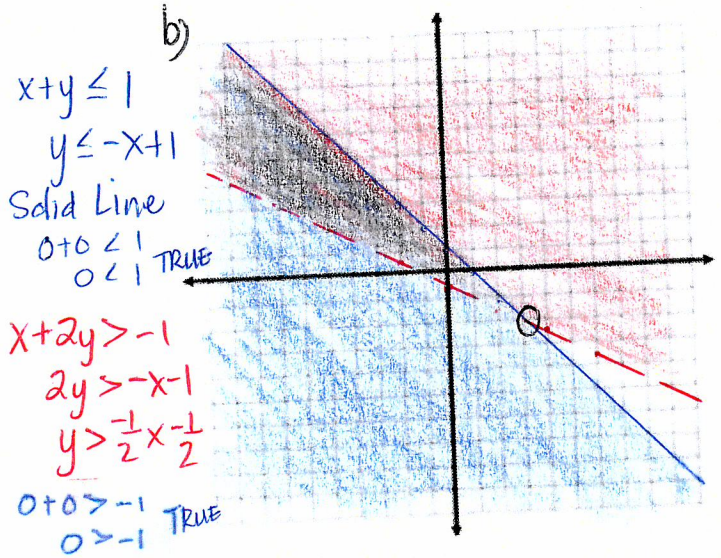
ii) environment = 12.6 ± 3.1%

9.5% – 15.7%

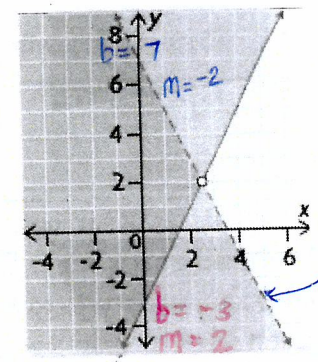
45. a)



b)



46.



$y < -2x + 7$

check (0,0)

$0 < -2(0) + 7$

$0 < 7$

dashed

Solid

$y \geq 2x - 3$

check (0,0)

$0 \quad 2(0) - 3$

$0 \geq -3$

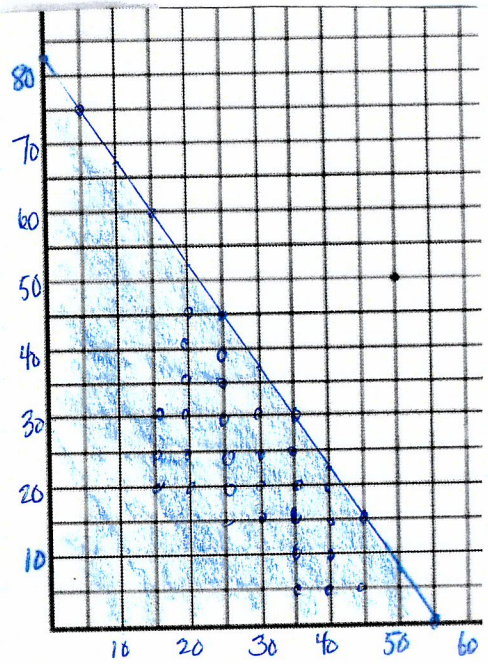
47. x = rectangular table (12 people)

a) y = circular table (8 people)

$12x + 8y \leq 660$

$y \leq -\frac{3}{2}x + 82.5$

restricted to whole #'s. So stipple the boundary line & solid region



b) as close to the same # of tables as possible (30, 30) would work but it wouldn't maximize seating. (33, 33) would maximize.

33 (12) + 33 (8) =

396 + 264 = 660.

- Nick's Soup -

48. x = peppers $x, y \in R$

a) y = tomatoes

$x \geq 2y$

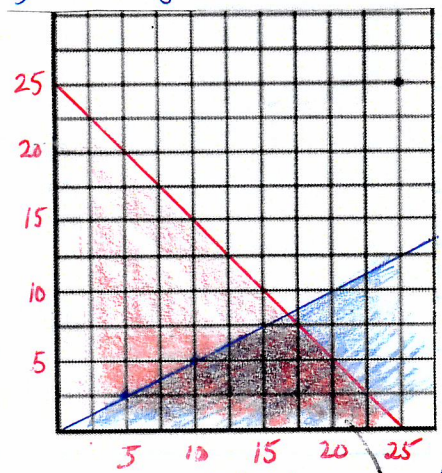
$x + y \leq 25$

$y \leq -x + 25$

$2y \leq x$

$y \leq \frac{1}{2}x$

b)



combinations (peppers, tomatoes) in the sol'n region

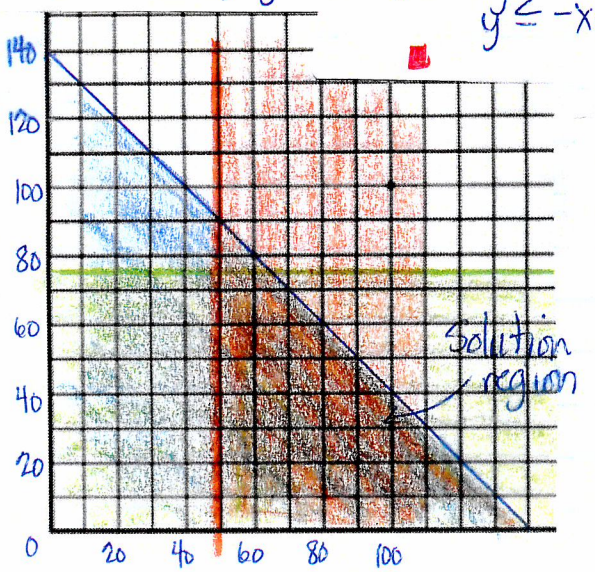
(10, 5) (15, 5) (20, 5)

49. a) x = ribbon flowers $x, y \in W$

y = crepe paper roseffes

$x \geq 50$, $y \leq 75$, $x + y \leq 140$

$y \leq -x + 140$



49. Continued..... Minimizing Time.

Objective Function: $T = 6x + 9y$

Vertices: (50, 0) (50, 75) (65, 75) (140, 0)

$T = 6(50) + 9(0)$ $T = 6(50) + 9(75)$ $T = 6(65) + 9(75)$

$T = 300$ min $T = 975$ min $T = 1065$ min

↑

minimum Time would be 50 ribbon flowers and no crepe paper roseffes.

It would take 300 minutes or 5 hours.

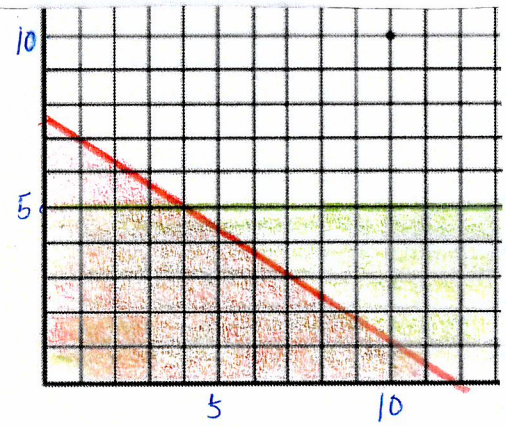
50. x = vans y = minibuses $x, y \in W$

$y \leq 5$, $10x + 16y \leq 120$

$16y \leq -10x + 120$

$y \leq -\frac{5}{8}x + 7.5$

objective Function: $C = 550x + 730y$



Vertices: vans minibuses

(0, 5) (4, 5) (12, 0)

$550(0) + 730(5)$ $550(4) + 730(5)$ $550(12) + 730(0)$

$C = 3650$ $C = 5850$ $C = 6600$

maximum value comes with 12 minivans and no minibuses $6600. transporting 120 people.

51. n = narrow boards $\quad n, w \in W$ (convert 50 y x 36 ✓)

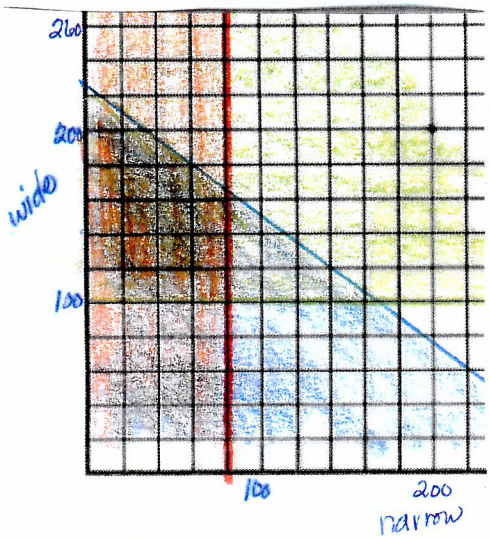
w = wide boards

$w \geq 100$, $n \leq 80$, $6n + 8w \leq 1800$

$8w \leq -6n + 1800$

$w \leq -\frac{3}{4}n + 225$

OBJ FUN: $3.56n + 4.36w = C$



Vertices: (narrow, wide)

(0,100)	(0,225)
$436.00	$981.00
(80,100)	(80,165)
$720.00	$1004.20

see pg 339 for more

Minimum Cost: 0 narrow / 100 wide = $436.00

Maximum Cost: 80 narrow / 165 wide = $1004.20

52. x = job #1 pays 8.75/hr $\quad x, y \in W$

y = job #2 pays 9.00/hr

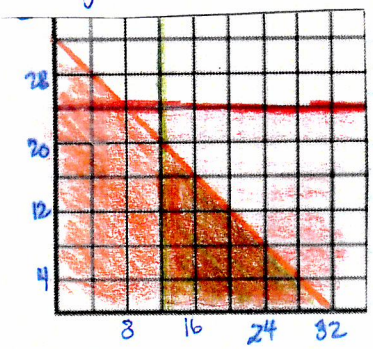
E = earnings

$x + y \leq 32$

$x \geq 12$

$y \leq 24$

OBJ FUN: $E = 8.75x + 9.00y$



Vertices

(12,0) $105.00

(32,0) $280.00

(12,20) 105 + 180 $285.00

Maximum Earnings

12 hrs @ 8.75/hr

20 hrs @ 9.00/hr.

53. a) $x^2 - 8x = f(x)$

$x(x-8) = f(x)$

$x = 0 \quad x - 8 = 0$

$x = 8$

- x intercepts are 0 and 8

$\frac{0+8}{2} = \frac{8}{2} \rightarrow 4$

- axis of symmetry $x = 4$

vertex lies on the axis of symmetry

$4^2 - 8(4) =$

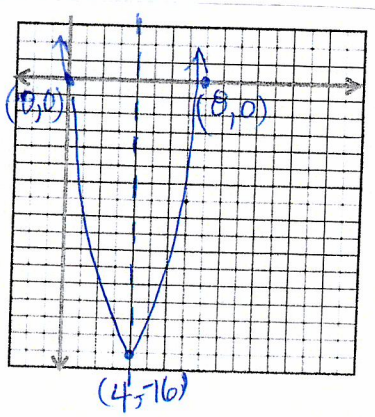
$16 - 32 = -16$

- vertex $(4, -16)$

y-intercept $x = 0$

$0^2 - 8(0) = 0$

- y intercept $\rightarrow (0,0)$



Domain $x \in \mathbb{R}$

Range $y \geq -16$

53. b) $y = x^2 + 2x - 15$

$y = (x+5)(x-3)$

$x+5=0 \quad x-3=0$

$x=-5 \quad x=+3$

- axis of symmetry $x=-1$

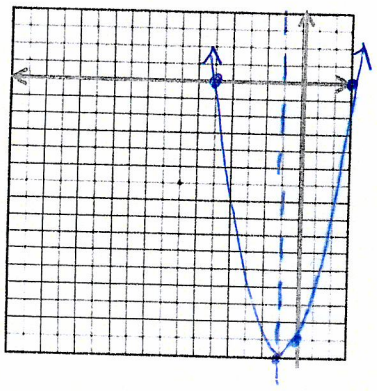
$\frac{(-5+3)}{2} = \frac{-2}{2} = -1$

- $y = (-1)^2 + 2(-1) - 15$

$y = 1 - 2 - 15$

$y = -16$

- vertex: $(-1, -16)$
- y intercept = -15



Domain $x \in \mathbb{R}$

Range $y \geq -16$

54. $y = -2(x-3)^2 + 8$

a) vertex = $(3, 8)$

axis of symm $x = 3$

y intercept = $-2(0-3)^2 + 8$

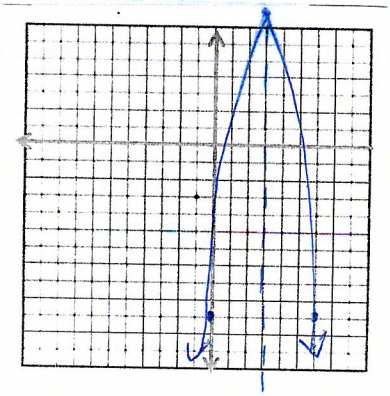
$-2(-3)^2 + 8$

$-2(9) + 8$

$= -10$

$x \in \mathbb{R}$

$y \leq 8$



54. b) $g(x) = .5(x+2)^2 - 1$

vertex = $(-2, -1)$

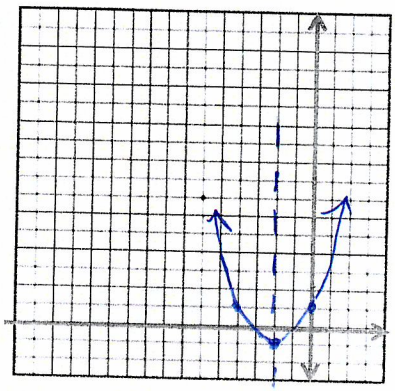
axis of symmetry $x = -2$

y int = $.5(0+2)^2 - 1$

$.5(4) - 1$

$2 - 1$

$(0, 1)$



$D = x \in \mathbb{R}$

$R = y \geq -1$

55. $f(x) = 4x^2 + 24x + 31$

axis of symmetry $x = 23$

a) opens up because a is positive

b) the vertex lies on the axis of symm.

So plug it in... $4(-3)^2 + 24(-3) + 31$

$4(9) + (-72) + 31$

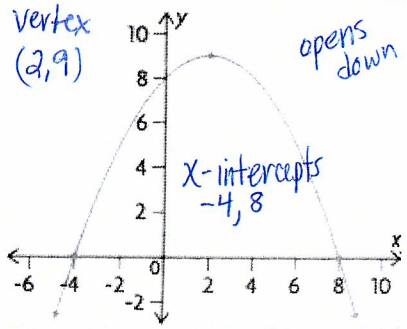
$36 + (-72) + 31 = -5$

vertex = $(-3, -5)$

c) a minimum value because it opens up.

56. vertex $(2, 9)$

opens down



$y = a(x-r)(x-5)$

$9 = a(2-(-4))(2-8)$

$9 = a(6)(-6) \rightsquigarrow \frac{9}{-36} = \frac{-36a}{-36}$

$y = \frac{-1}{4}(x+4)(x-8)$ OR $y = -\frac{1}{4}x^2 + x + 8$

57. Determine the Roots (aka: the x-intercepts)

a) -3 and 3 } just by looking at graph.

b) $-\frac{1}{2}$ and 3 }

58. a) $(x-5)(2x+1)=0$

$x-5=0 \quad 2x+1=0$

$x=5 \quad x=-\frac{1}{2}$

b) $x^2-4x-32=0$

$(x-8)(x+4)=0$

$x=8 \quad x=-4$

c) $3x^2-10x-8=0$

$(3x+2)(x-4)=0$

$x=-\frac{2}{3} \quad x=4$

$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

d) $x^2-6-10=0$! can't factor so use Quad Formula

$a=1 \quad b=-6 \quad c=-10$

$\frac{-(-6)\pm\sqrt{36-4(1)(-10)}}{2} \Rightarrow \frac{6\pm\sqrt{76}}{2}$

$=\frac{6\pm 2\sqrt{19}}{2} \Rightarrow 3\pm\sqrt{19}$

e) $2(x-3)^2-8=0$

$2(x^2-6x+9)-8=0$

$2x^2-12x+18-8=0$

$2x^2-12x+10=0$

$2(x^2-6x+5)=0$

$(x-5)(x-1)=0$

$x=5 \quad x=1$

f) $1.5x^2-6.1x+1.1=0$ (a) (b) (c)

$\frac{-(-6.1)\pm\sqrt{(-6.1)^2-4(1.5)(1.1)}}{2(1.5)}$

$\frac{6.1\pm\sqrt{37.21-6.6}}{3}$

$\frac{6.1\pm\sqrt{30.61}}{3}$

$\frac{6.1+5.53}{3}=3.88$

$\frac{6.1-5.53}{3}=.19$

59. $h(t)=4500-5t^2$

a) $h=4500-5(5)^2$

$h=4500-125$

$h=4375$ m

b) $1500=4500-5t^2$

$5t^2-3000=0$

$5(t^2-600)=0$

$\sqrt{t^2}=\sqrt{600}$

$t=24.5$ s

60. y-int $=(0,-4)$ Vertex $(3,-7)$

$y=a(x-h)^2+k$

$-4=a(0-(-3))^2+(-7)$

$-4=a(9)-7$

$\frac{3}{9}=\frac{9a}{9}$ $\quad a=\frac{1}{3}$

$y=\frac{1}{3}(x+3)^2-7$

$y=\frac{1}{3}(x^2+6x+9)-7$

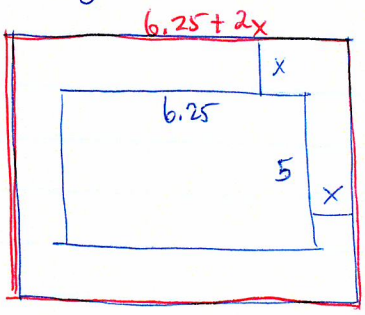
$y=\frac{1}{3}x^2+2x+3-7$

$y=\frac{1}{3}x^2+2x-4$

61.

Area of path = Big $\square$ − Small $\square$

$A=l\times w$



amount of crushed rock

$(6.25+2x)(5+2x)-(6.25)(5)=6$ m²

$31.25+10x+12.5x+4x^2-31.25=6$

$4x^2+22.5x-6=0$

$a=4$

$b=22.5$

$c=-6$

$\frac{-22.5\pm\sqrt{(22.5)^2-4(4)(-6)}}{2(4)}$

$\frac{-22.5\pm\sqrt{602.25}}{8}$

$\frac{-22.5\pm 24.5}{8}$

$\frac{2}{8}=.25$

$\frac{-47}{8}=-5.875$ impossible

The border should be .25 m wide.