

Math Precalc 20
Chapter 6 Review

#1. Simplify and determine the non-permissible values. (2 marks each = 8 marks)

a) $\frac{-35a^2b^3c^4}{40abc^7} = \frac{-7a^{2-1}b^{3-1}c^{4-7}}{8} = \frac{-7ab^2c^{-3}}{8} = \frac{-7ab^2}{8c^3}$

$a \neq 0$
 $b \neq 0$
 $c \neq 0$

b) $\frac{6x^2 - 8x}{4x} = \frac{2x(3x-4)}{2 \cdot 2x} = \frac{3x-4}{2}$

$x \neq 0$

c) $\frac{m^2 + 2m - 3}{3m^2 + 9m} = \frac{(m+3)(m-1)}{3m(m+3)} = \frac{m-1}{3m}$

$m \neq 0$
 $m \neq -3$

d) $\frac{2x^3 - 28x^2 - 102x}{18x - 2x^3}$

$\frac{2x(x^2 - 14x - 51)}{2x(9 - x^2)} = \frac{2x(x-17)(x+3)}{2x(3-x)(3+x)}$

$\frac{x-17}{3-x}$ or $\frac{x-17}{-(x-3)}$

$x \neq 0$
 $x \neq -3$
 $x \neq 3$

#2. Simplify the following. (3 marks each = 9 marks)

a) $\frac{(2m)^2}{5n} \times \frac{10m}{8n} \div \frac{15m}{(4n)^2}$

$\frac{4m^2 \times 10m \times 16n^2}{5n \times 8n \times 15m} = \frac{16m^2}{15}$

OR $\frac{640m^3n^2}{600m^2n^2} = \frac{16m}{15}$

b) $\frac{x^2 - 3x - 4}{2x + 2} \times \frac{4x}{16 - x^2}$

$\frac{(-1)(x-4)(x+1) \cdot 4x}{2(x+1) \cdot (4-x)(4+x)} = \frac{(-x+4)(2x)}{(4-x)(4+x)} \Rightarrow \frac{2x}{-1(4+x)}$

could also look like: $\frac{-2x}{(x+4)}$

c) $\frac{2x^2 + 5xy + 2y^2}{3x^2 - 8xy - 3y^2} \times \frac{x^2 - 9y^2}{x^2 - 4y^2} \div \frac{3x^2 + 11xy + 6y^2}{2x^2 - 3xy - 2y^2}$

$\frac{(x+2y)(2x+y) \cdot (x-3y)(x+3y) \cdot (x-2y)(2x+y)}{(x-3y)(3x+y) \cdot (x-2y)(x+2y) \cdot (x+3y)(3x+2y)} = \frac{(2x+y)^2}{(3x+y)(3x+2y)}$

#3. Simplify the following:

(a, b = 6 marks, c = 4 marks, Total = 10 marks)

LCD = 18xy

a) $\left(\frac{6x-11y}{9x} + \frac{3x-16y}{6y}\right) \frac{3x}{3x}$

$\frac{2y(6x-11y) + 3x(3x-16y)}{18xy} = \frac{12xy - 22y^2 + 9x^2 - 48xy}{18xy} = \frac{9x^2 - 36xy - 22y^2}{18xy}$

b) $\frac{7x}{x^2 - x - 12} - \frac{4x}{x^2 + 2x - 3}$

$\frac{(x-1) \cdot 7x}{(x-1)(x-4)(x+3)} - \frac{4x \cdot (x-4)}{(x+3)(x-1)(x-4)}$

$\frac{7x(x-1) - 4x(x-4)}{(x-1)(x-4)(x+3)} = \frac{7x^2 - 7x - 4x^2 + 16x}{(x-1)(x-4)(x+3)} = \frac{3x^2 + 9x}{(x-1)(x-4)(x+3)} = \frac{3x(x+3)}{(x-1)(x-4)(x+3)} = \frac{3x}{(x-1)(x-4)}$

$$\frac{(x+1)}{(x+1)} \cdot \frac{2x-1+\frac{3x}{x+1}}{x+1}$$

$$\frac{(x+1)3x - \frac{x}{x+1}}{x+1}$$

$$\frac{(x+1)(2x-1) + \frac{3x}{x+1}}{x+1}$$

$$\frac{3x(x+1) - \frac{x}{x+1}}{x+1}$$

$$\frac{2x^2+x-1+\frac{3x}{x+1}}{x+1}$$

$$\frac{3x^2+3x-x}{x+1}$$

$$\frac{2x^2+4x-1}{(x+1)} \cdot \frac{(x+1)}{3x^2+2x}$$

$$\frac{2x^2+4x-1}{3x^2+2x}$$

$$\frac{(x+3)(x-2)}{x+3} \cdot \frac{2}{x-2} = \frac{15}{(x+3)(x-2)}$$

$$7(x-2) - 2(x+3) = 15$$

$$7x-14-2x-6=15$$

$$5x-20=15$$

$$5x=35 \text{ or } 5x-35=0$$

$$\frac{5}{5} = \frac{35}{5} \quad 5(x-7)=0$$

$$x=7 \quad x=7$$

#4. Solve. (3 marks each = 12 marks)

a) $\frac{5x}{5} \left(\frac{x+15}{5} \right) = \frac{5x}{5} \left(\frac{2}{x} \right) + \frac{5x}{5} \left(\frac{x+1}{5} \right)$ LCD = 5x

$$x(x+15) = 10 + x(x+1)$$

$$x^2+15x = 10+x^2+x$$

$$14x = 10$$

$$\frac{14x}{14} = \frac{10}{14}$$

$$x = \frac{5}{7}$$

#5. The sum of two numbers is 25. The sum of their reciprocals is $\frac{1}{4}$. Determine the two numbers. $x+y=25$

Let $x = 1st \#$ and $25-x = 2nd \#$

(4 marks)

$$\left(\frac{1}{x} + \frac{1}{25-x} = \frac{1}{4} \right) \times (25-x)(4)$$

$$(25-x)(4) + 4(x) = x(25-x)$$

$$100 - 4x + 4x = 25x - x^2$$

$$100 - 25x + x^2 = 0$$

$$(20-x)(5-x) = 0$$

$$20-x=0 \quad 5-x=0$$

$$20=x \quad 5=x$$

The two numbers are 5 and 20.

now multiply the entire equation by the complete LCD

b) $\frac{9x^2}{x^2-25} = \frac{4x}{x-5} + \frac{x}{x+5}$

$$x^2-25 \left(\frac{9x^2}{x^2-25} \right) = x^2-25 \left(\frac{4x}{x-5} \right) + x^2-25 \left(\frac{x}{x+5} \right)$$

$$9x^2 = 4x(x+5) + x(x-5)$$

$$9x^2 = 4x^2 + 20x + x^2 - 5x$$

$$4x^2 - 15x = 0$$

$$x(4x-15) = 0$$

$$x=0 \quad 4x-15=0$$

$$x = \frac{15}{4}$$

#6. The sum of the reciprocals of two consecutive integers is $\frac{11}{30}$. What are the integers?

Let $x = 1st \text{ integer}$
 $x+1 = next \text{ consec. integer}$

their reciprocals $\frac{1}{x} + \frac{1}{x+1}$

$$\frac{1}{x} + \frac{1}{x+1} = \frac{11}{30} \quad \text{LCD} = 30(x)(x+1)$$

$$30(x)(x+1) \left(\frac{1}{x} \right) + 30(x)(x+1) \left(\frac{1}{x+1} \right) = 30(x)(x+1) \left(\frac{11}{30} \right)$$

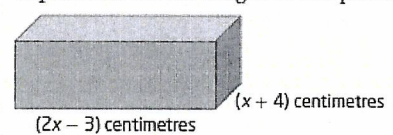
$$30x + 30 + 30x = 11x^2 + 11x$$

$$0 = 11x^2 - 49x - 30$$

$$0 = (11x+6)(x-5)$$

so, 5 and 6. $x=5$
 $x = -\frac{6}{11}$ not an integer

#7. The volume of a rectangular prism is $(2x^3 + 5x^2 - 12x)$ cubic centimetres. If the length of the prism is $(2x - 3)$ centimetres and its width is $(x + 4)$ centimetres, what is an expression for the height of the prism? (3 marks)



$$2x^3 + 5x^2 - 12x = (2x-3)(x+4)(h)$$

$$\frac{2x^3 + 5x^2 - 12x}{(2x-3)(x+4)} = h$$

$$\frac{x(2x^2 + 5x - 12)}{(2x-3)(x+4)} = h$$

$$\frac{x(x+4)(2x-3)}{(2x-3)(x+4)} = h$$

height = x

c) $x - \frac{1}{x+4} = -4$

$$x(x+4) - \left(\frac{1}{x+4} \right) x+4 = -4(x+4)$$

$$x^2+4x-1 = -4x-16$$

$$x^2+8x+15=0$$

$$(x+5)(x+3)=0$$

$$x=-5 \quad x=-3$$