Math Pre-Calc 20 Midterm #1 (Solutions)

Chp 3 Quadratic Functions

#1. Find the vertex of each quadratic:

a) $y = 3x^2$

vertex is (0, 0)

b) $y + 3 = -\frac{1}{2}x^2$

vertex is (0, -3)

c) $y = (x + 1)^2 + 2$

vertex is (-1, 2)

#2. Write each of the following in vertex-graphing form by completing the square:

a) $y = x^2 + 4x$

$y = x^2 + 4x + 4 - 4$

$y = (x + 2)^2 - 4$

b) $y = x^2 + x - 1$

$y + 1 = x^2 + 1x$

$y + 1 + \frac{1}{4} = x^2 + x + \frac{1}{4}$

$y + 1 + \frac{1}{4} = (x + \frac{1}{2})^2$

$y = (x + \frac{1}{2})^2 - \frac{5}{4}$

c) $y = -3x^2 + 12x - 2$

$y = -3x^2 + 12x$

$y + 2 = -3(x^2 - 4x)$

$y + 2 - 12 = -3(x^2 - 4x + 4)$

$y - 10 = -3(x^2 - 4x + 4)$

$y = -3(x - 2)^2 + 10$

#3. Answer the following questions for each quadratic function:
a) vertex b) equation of the axis of symmetry c) concavity (faces up or down)
d) maximum or minimum value e) domain and range f) x and y intercepts
g) sketch the graph

i) $y = -3(x + 2)^2 + 3$

Vertex is (-2, 3)
Eqn of A.O.S. is $x = -2$
Faces Down (a is neg)
Max Value of 3
Domain: $x \in R$
Range: $y \leq 3$

x-intercepts

$0 = -3(x + 2)^2 + 3$

$-3 = -3(x + 2)^2$

$1 = (x + 2)^2$

$\pm\sqrt{1} = x + 2$

$\pm 1 = x + 2$

$1 = x + 2$ $\quad$ $-1 = x + 2$

$-1 = x$ $\quad$ $-3 = x$

x ints are {-1, -3}

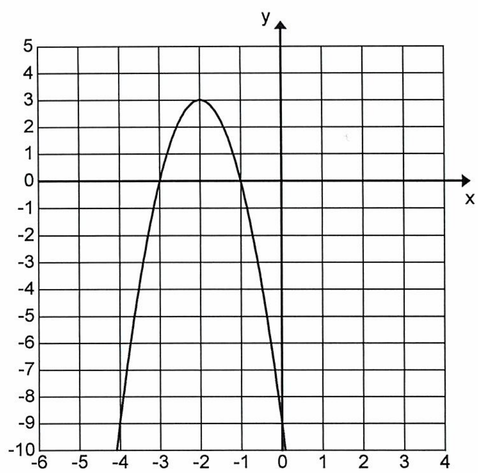
y-intercepts

$y = -3(0 + 2)^2 + 3$

$y = -3(4) + 3$

$y = -9$

y int is -9



ii) $y = x^2 + 4x + 3$

Complete the square:

$y = x^2 + 4x + 4 - 4 + 3$

$y = (x + 2)^2 - 1$

(or use $p = -\frac{b}{2a}$)

Vertex is (-2, -1)
Eqn of A.O.S. is $x = -2$
Faces Up (a is pos)
Min Value of -1
Domain: $x \in R$
Range: $y \geq -1$

x-intercepts

$0 = x^2 + 4x + 3$

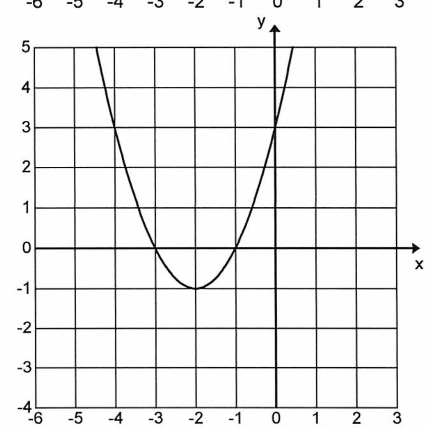
$0 = (x + 3)(x + 1)$

x ints are {-3, -1}

y-intercepts

$y = 0^2 + 4(0) + 3$

y int is 3



#4. Write a quadratic equation in vertex graphing form for each of the following:

a) a = 2 vertex is (-1, 2)

$y = a(x - p)^2 + q$

$y = 2(x + 1)^2 + 2$

b) vertex is (3, 2) and passes through the point (2, -1)

$y = a(x - p)^2 + q$ p=3, q=2, x=2, y=-1

$-1 = a(2 - 3)^2 + 2$

$-1 = a(1) + 2$

$-3 = a$ **$y = -3(x - 3)^2 + 2$**

#5. Write the new equation of the parabola $y = x^2$ after the following: (3 marks)

a) a horizontal translation 2 units to the left and a vertical translation 1 unit up

$y = a(x - p)^2 + q$ a=1, p=-2, q= 1 **$y = (x + 2)^2 + 1$**

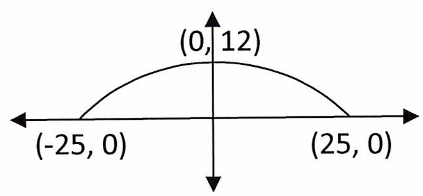
b) a vertical translation 3 units down and a reflection across the x-axis

$y = a(x - p)^2 + q$ a=-1, p=0, q= -3 **$y = -1x^2 - 3$**

c) a multiplication of the y-values by -2 and then a horizontal translation 1 unit to the right

$y = a(x - p)^2 + q$ a=-2, p=1, q= 0 **$y = -2(x - 1)^2$**

#6. A bridge has the shape of a parabola. Its width is 50m and its height is 12m. Find the quadratic equation for this bridge.



$y = a(x - p)^2 + q$ p=0, q=12, x=25, y=0

$0 = a(25 - 0)^2 + 12$

$0 = a(625) + 12$

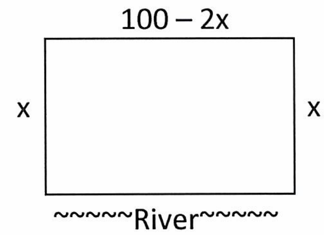
$-12 = 625a$ $a = -\frac{12}{625}$ $y = -\frac{12}{625}x^2 + 12$

#7. The height, "h", in metres, of a flare "t" seconds after it is fired into the air is given by the equation $h(t)=-4.9t^2 + 61.25t$. At what height is the flare at its maximum height? How many seconds after being shot does this occur?

$p = -\frac{b}{2a} = -\frac{61.25}{2(-4.9)} = 6.25$ $q = -4.9(6.25)^2 + 61.25(6.25) = 191.4$ Vertex is (6.25, 191.4)

Max height is at 191.4m. It happens 6.25 seconds after being shot.

#8. A farmer has 100m of fencing material to enclose a rectangular field adjacent to a river. No fencing is required along the river. Find the dimensions of the rectangle that will make its area a maximum. What is the maximum Area? (Hint: a diagram of the situation is given below)



$A = x(100 - 2x)$
$A = 100x - 2x^2$ or $A = -2x^2 + 100x$

$$p = -\frac{b}{2a} = -\frac{100}{2(-2)} = 25$$

$q = -2(25)^2 + 100(25) = 1250$ Vertex is (25, 1250)

$100 - 2(25) = 50$ So the rectangle is 25m by 50m. The maximum area is $1250m^2$.

Chp 4 Quadratic Equations

#1. Solve the quadratic equations by factoring:

a) $3x^2 - 36x = 0$

$3x(x - 12) = 0$
$x = 0 \quad x = 12$
$\{0, 12\}$

b) $2x^2 - 7x - 15 = 0$

$(2x + 3)(x - 5) = 0$
$\left\{-\frac{3}{2}, 5\right\}$

c) $6x^2 - 11x + 3 = 24$

$6x^2 - 11x + 3 = 24$
$6x^2 - 11x - 21 = 0$
$(6x + 7)(x - 3) = 0$
$\left\{-\frac{7}{6}, 3\right\}$

#2. Solve the quadratic equations by completing the square: (Write answers in Exact Form)

a) $x^2 - 6x + 5 = 0$

$x^2 - 6x = -5$
$x^2 - 6x + 9 = -5 + 9$
$(x - 3)^2 = 4$
$x - 3 = \pm\sqrt{4}$
$x - 3 = \pm 2$
$x - 3 = 2$ or $x - 3 = -2$
$x = 5 \qquad x = 1$
$\{5, 1\}$

b) $x^2 + 4x + 1 = 0$

$x^2 + 4x = -1$
$x^2 + 4x + 4 = -1 + 4$
$(x + 2)^2 = 3$
$x + 2 = \pm\sqrt{3}$
$x = -2 \pm \sqrt{3}$
$\{-2 \pm \sqrt{3}\}$

c) $3x^2 - x - 2 = 0$

$3x^2 - x = 2$
$x^2 - \frac{1}{3}x = \frac{2}{3}$
$x^2 - \frac{1}{3}x + \frac{1}{36} = \frac{2}{3} + \frac{1}{36}$
$\left(x - \frac{1}{6}\right)^2 = \frac{25}{36}$
$x - \frac{1}{6} = \pm\sqrt{\frac{25}{36}}$
$x = \frac{1}{6} \pm \frac{5}{6} \qquad \left\{\frac{6}{6}, -\frac{4}{6}\right\} = \left\{1, -\frac{2}{3}\right\}$

#3. Solve the quadratic equations using the quadratic formula: (Write answers in Exact Form)

a) $x^2 + 4x - 96 = 0$ a =1, b=4, c=-96

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-96)}}{2(1)} = \frac{-4 \pm \sqrt{400}}{2} = \frac{-4 \pm 20}{2} = -2 \pm 10$$ {-12, 8}

b) $3x^2 = 4$ (Hint: Same as $3x^2 - 0x - 4 = 0$) a=3, b=0, c=-4

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{0 \pm \sqrt{(0)^2 - 4(3)(-4)}}{2(3)} = \frac{\pm\sqrt{48}}{6} = \frac{\pm 4\sqrt{3}}{6} = \frac{\pm 2\sqrt{3}}{3}$$ $\left\{\frac{\pm 2\sqrt{3}}{3}\right\}$

#4. Find the zeros of the function $f(x) = x^2 - 10x + 16$.

$0 = x^2 - 10x + 16$

$0 = (x - 8)(x - 2)$

$x = 8$ $x = 2$ The zero's are 8 and 2. {Note: The zeros are the same as x-intercepts!}

#5. Find the quadratic equation with the roots of $\left\{\frac{1}{2}, -\frac{2}{3}\right\}$

$(2x - 1)(3x + 2) = 0$

$6x^2 + x - 2 = 0$

#6. Find the discriminant and state the nature of the roots:

a) $x^2 - 4x - 5 = 0$

a=1 b=-4 c=-5

Discr = $b^2 - 4ac$

Discr = $(-4)^2 - 4(1)(-5)$

Discr = 16 + 20 = 36

So there are **2** roots.

b) $x^2 = -9$

$x^2 + 9 = 0$ a=1 b=0 c=9

Discr = $b^2 - 4ac$

Discr = $(0)^2 - 4(1)(9)$

Discr = 0 – 36 = -36

So there are **0** roots.

c) $x^2 + 2x + 1 = 0$

a=1 b=2 c=1

Discr = $b^2 - 4ac$

Discr = $(2)^2 - 4(1)(1)$

Discr = 4 – 4 = 0

So there is **1** root.

#7. The hypotenuse of a right triangle is 13. If the sum of the legs is 17, find the legs.
(Hint: Let one leg be x and the other is therefore 17-x...since the sum is 17.)

$a^2 + b^2 = c^2$

$x^2 + (17 - x)^2 = 13^2$ $(17 - x)(17 - x) = 289 - 17x - 17x + x^2$

$x^2 + 289 - 34x + x^2 = 169$

$2x^2 - 34x + 120 = 0$

$2(x^2 - 17x + 60) = 0$

$(x - 12)(x - 5) = 0$

$x = 12$ $x = 5$ The legs are 5 and 12.

#8. If $h(t) = 5t^2 - 30t + 45$, find t when h = 20. (Hint: $20 = 5t^2 - 30t + 45$)

$20 = 5t^2 - 30t + 45$
$0 = 5t^2 - 30t + 25$
$0 = 5(t^2 - 6t + 5)$
$0 = (t - 5)(t - 1)$
t= 5 t = 1 {5, 1}

Chp 8 Systems

#1. Solve by graphing. Give approximate solutions if needed. Verify your solutions.

$y = \frac{1}{2}x + 2$
$y + x^2 + 2x = 8$

$y = -x^2 - 2x + 8$

$$p = -\frac{b}{2a} = \frac{-(-2)}{2(-1)} = -1$$

$q = -(-1)^2 - 2(-1) + 8 = 9$

Vertex is at (-1, 9)

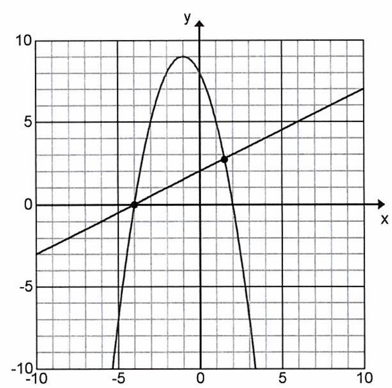
Check (-4,0):
$y = \frac{1}{2}x+2$
$0 = \frac{1}{2}(-4)+2$
$0 = -2+2$
0 = 0 yes

$y + x^2 + 2x = 8$
$0 + (-4)^2 + 2(-4) = 8$
0 + 16 – 8 = 8
8 = 8 yes

Check (1.5,2.7):
$y = \frac{1}{2}x+2$
$2.7 = \frac{1}{2}(1.5) + 2$
2.7 = .75 + 2
2.7 = 2.75 yes, close

$y + x^2 + 2x = 8$
$2.7 + (1.5)^2 + 2(1.5) = 8$
2.7 + 2.25 + 3 = 8
7.95 = 8 yes, close

Solutions: **{(-4,0) (1.5,2.7)}** approximately



#2. Solve algebraically. Verify your solutions.

$y = 3x + 1$
$y = 6x^2 + 10x - 4$

Substitute 3x + 1 in for y in the 2nd equation:

$3x + 1 = 6x^2 + 10x - 4$
$0 = 6x^2 + 7x - 5$
$0 = (2x - 1)(3x + 5)$

$x = \frac{1}{2}$ $\qquad x = -\frac{5}{3}$

substitute x to find y values

$y = 3x + 1$ $\qquad y = 3x + 1$

$y = 3\left(\frac{1}{2}\right) + 1$ $\qquad y = 3\left(-\frac{5}{3}\right) + 1$

$y = \frac{5}{2}$ $\qquad y = -4$

Solutions: $\left\{\left(\frac{1}{2}, \frac{5}{2}\right), \left(-\frac{5}{3}, -4\right)\right\}$

Check: $\left(\frac{1}{2}, \frac{5}{2}\right)$

$y = 3x + 1$

$$\frac{5}{2} = 3\left(\frac{1}{2}\right) + 1$$
$$\frac{5}{2} = \frac{3}{2} + \frac{2}{2}$$
$$\frac{5}{2} = \frac{5}{2}$$

$y = 6x^2 + 10x - 4$

$$\frac{5}{2} = 6\left(\frac{1}{2}\right)^2 + 10\left(\frac{1}{2}\right) - 4$$
$$\frac{5}{2} = 6\left(\frac{1}{4}\right) + 5 - 4$$
$$\frac{5}{2} = \frac{3}{2} + 1$$
$$\frac{5}{2} = \frac{5}{2}$$

Check: $\left(-\frac{5}{3}, -4\right)$

$y = 3x + 1$

$$-4 = 3\left(-\frac{5}{3}\right) + 1$$
$$-4 = -5 + 1$$
$$-4 = -4$$

$y = 6x^2 + 10x - 4$

$$-4 = 6\left(-\frac{5}{3}\right)^2 + 10\left(-\frac{5}{3}\right)$$
$$-4 = 6\left(\frac{25}{9}\right) - \frac{50}{3} - 4$$
$$-4 = \frac{50}{3} - \frac{50}{3} - 4$$
$$-4 = -4$$

#3. Solve algebraically. Verify your solutions.

$x^2 + y - 3 = 0$
$x^2 - y + 1 = 0$ Add both together to eliminate the y terms

$2x^2 - 2 = 0$
$2x^2 = 2$
$x^2 = 1$
$x = \pm 1$

You could also use substitution to solve this problem!

substitute x to find y values

x = 1	x = -1
$x^2 + y - 3 = 0$	$x^2 + y - 3 = 0$
$(1)^2 + y - 3 = 0$	$(-1)^2 + y - 3 = 0$
$1 + y - 3 = 0$	$1 + y - 3 = 0$
$y = 2$	$y = 2$

Solutions: {(1,2) , (-1,2)}

Check: (1, 2)

$x^2 + y - 3 = 0$
$(1)^2 + 2 - 3 = 0$
$1 + 2 - 3 = 0$
$0 = 0$

$x^2 - y + 1 = 0$
$(1)^2 - 2 + 1 = 0$
$1 - 2 + 1 = 0$
$0 = 0$

Check: (-1, 2)

$x^2 + y - 3 = 0$
$(-1)^2 + 2 - 3 = 0$
$1 + 2 - 3 = 0$
$0 = 0$

$x^2 - y + 1 = 0$
$(-1)^2 - 2 + 1 = 0$
$1 - 2 + 1 = 0$
$0 = 0$

#4. Solve algebraically. Verify your solutions.

$y = x^2 - 4x + 1$
$2y = -x^2 + 4x + 2$

substitute $x^2 - 4x + 1$ in for y in the 2nd equation:

$2(x^2 - 4x + 1) = -x^2 + 4x + 2$
$2x^2 - 8x + 2 = -x^2 + 4x + 2$
$3x^2 - 12x = 0$
$3x(x - 4) = 0$
$x = 0 \quad x = 4$

substitute x to find y values

x = 0	x = 4
$y = x^2 - 4x + 1$	$2y = -x^2 + 4x + 2$
$y = (0)^2 - 4(0) + 1$	$2y = -(4)^2 + 4(4) + 2$
$y = 1$	$2y = 2$
	$y = 1$

Solutions: {(0,1) , (4,1)}

Check: (0, 1)

$y = x^2 - 4x + 1$
$1 = (0)^2 - 4(0) + 1$
$1 = 1$

$2y = -x^2 + 4x + 2$
$2(1) = -(0)^2 + 4(0) + 2$
$2 = 2$

Check: (4, 1)

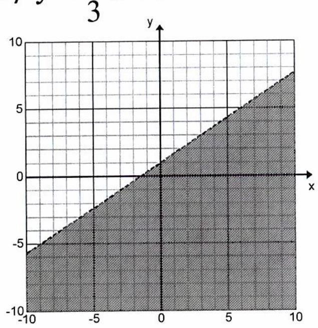
$y = x^2 - 4x + 1$
$1 = (4)^2 - 4(4) + 1$
$1 = 16 - 16 + 1$
$1 = 1$

$2y = -x^2 + 4x + 2$
$2(1) = -(4)^2 + 4(4) + 2$
$2 = -16 + 16 + 2$
$2 = 2$

Chp 9 Quadratic Inequalities

#1. Solve by graphing:

a) $y < \frac{2}{3}x + 1$



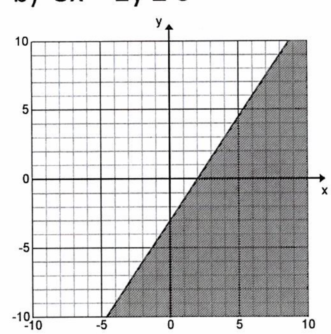
Slope is $\frac{2}{3}$ y-int: 1

Test Point: (0,0)

$0 < \frac{2}{3}(0) + 1$

$0 < 1$ true, so shade towards the pt (0,0)

b) $3x - 2y \geq 6$



$3x - 2y \geq 6$
$-2y \geq -3x + 6$
$\frac{-2y}{-2} \leq \frac{-3x}{-2} + \frac{6}{-2}$
$y \leq \frac{3}{2}x - 3$

Test Point: (0,0)

$0 \leq \frac{3}{2}(0) - 3$

$0 \leq -3$ false, so shade away from (0,0)

#2. Solve:

a) $x^2 + x - 12 < 0$

$(x + 4)(x - 3) < 0$ **zeros at -4 and 3**

Interval	x < -4	-4 < x < 3	x > 3
Test Point	-5	0	4
Substitution (Work Area)	$(-5)^2 + (-5) - 12$ 25-5-12 20-12 8	$0^2 + 0 - 12$ -12	$4^2 + 4 - 12$ 16 + 4 - 12 20 - 12 8
Result: + or –	+	-	+

Solution is {x/ -4 < x < 3, x∈R}

b) $x^2 > 5x$

$x^2 - 5x > 0$ $x(x - 5) > 0$ **zeros at 0 and 5**

Interval	x < 0	0 < x < 5	x > 5
Test Point	-1	1	6
Substitution (Work Area)	$(-1)^2 - 5(-1)$ 1 + 5 6	$(1)^2 - 5(1)$ 1 - 5 -4	$(6)^2 - 5(6)$ 36 - 30 6
Result: + or –	+	-	+

Solution is { x/x < 0 and x > 5, x∈R}

c) $x^2 - 3x + 6 < 2x$

$x^2 - 5x + 6 < 0$ $(x - 3)(x - 2) < 0$

zeros at 2 and 3

Interval	x < 2	2 < x < 3	x > 3
Test Point	-3	2.5	4
Substitution (Work Area)	$(-3)^2 -5(-3) + 6$ 9+15+6 30	$(2.5)^2 -5(2.5) + 6$ 6.25-12.5+6 -.25	$(4)^2 -5(4) + 6$ 16-20+6 2
Result: + or –	+	-	+

Solution is {x/ 2 < x < 3, x∈R}

d) $2x^2 < 3 - 5x$

$2x^2 + 5x - 3 < 0$ $(2x - 1)(x + 3) < 0$

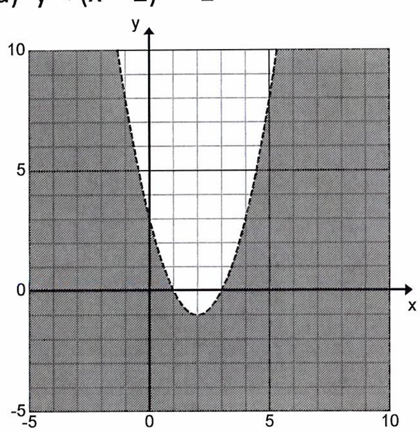
zeros at -3 and ½

Interval	x < -3	-3 < x < ½	x > ½
Test Point	-4	0	1
Substitution (Work Area)	$2(-4)^2 +5(-4) - 3$ 32-20-3 9	$2(0)^2 +5(0) - 3$ -3	$2(1)^2 +5(1) - 3$ 2+5-3 4
Result: + or –	+	-	+

Solution is {x/ -3 < x < ½, x∈R}

#3. Solve by graphing:

a) $y < (x - 2)^2 - 1$



Vertex: (2, -1) Use 1a/3a/5a to graph

Test Point: (0,0)

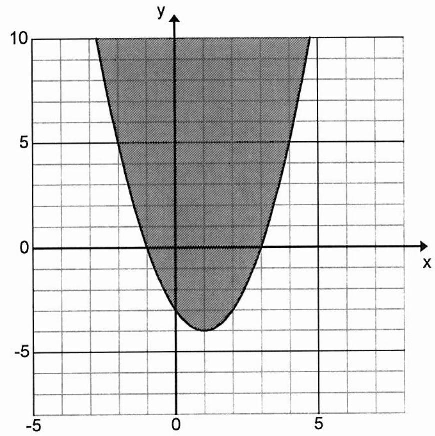
$y < (x - 2)^2 - 1$

$0 < (0 - 2)^2 - 1$

$0 < 4 - 1$

$0 < 3$ True, so shade towards pt (0,0)

b) $y + 3 \geq x^2 - 2x$



$y \geq x^2 - 2x - 3$

$$p = -\frac{b}{2a} = -\frac{-2}{2(1)} = \frac{2}{2} = 1$$

$q = (1)^2 - 2(1) - 3 = -4$

Vertex: (1, -4) Use 1a/3a/5a to graph
or use x-intercepts: $(x - 3)(x + 1)$
x-intercepts: 3 and -1

Test Point: (0,0)

$y \geq x^2 - 2x - 3$
$0 \geq (0)^2 - 2(0) - 3$
$0 \geq -3$ True, so shade towards pt (0,0)